Quantum Tunneling: From Bubbles of Nothing to String Theory Dynamical Cobordism

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String Phenomenology 2022, Liverpool

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References

Tunneling potential method [Espinosa, ’18]

Short review here

Application to the study of String Dynamical Cobordisms [Angius, Calderón, Delgado, JH, Uranga, ’22]

Roberta’s talk

Application to the study of Bubbles of Nothing [Blanco-Pillado, Espinosa, JH, Sousa, in progress]

This talk

Motivation

Bubbles of Nothing are important in…

…assessing the stability of vacua, in particular our universe

…studies of the behaviour of models with compactified dimensions

…String Theory Dynamical Cobordism

The Tunneling Potential Method

[Espinosa, '18]

$$S = \int d^{d+1}x \sqrt{-g} \left(\frac{1}{2\kappa} R - \frac{1}{2} (\partial\phi)^2 - V(\phi) \right)$$

Einstein gravity + scalar

+

$$ds^2 = d\xi^2 + \rho(\xi)^2 ds_d^2$$

Codimension-1 metric, with internal curvature

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Codimension-1 metric, with internal curvature

↓

$$V_t(\phi) \equiv V(\phi) - \frac{1}{2}\left(\frac{d\phi}{d\xi}\right)^2$$

$$((1+d)V_t' - dV')V_t' = 2d(V_t - V)\left[V_t'' + \frac{2\kappa}{d-1}(dV - (d-1)V_t)\right]$$

EoM of the system

Coleman-de Luccia tunneling

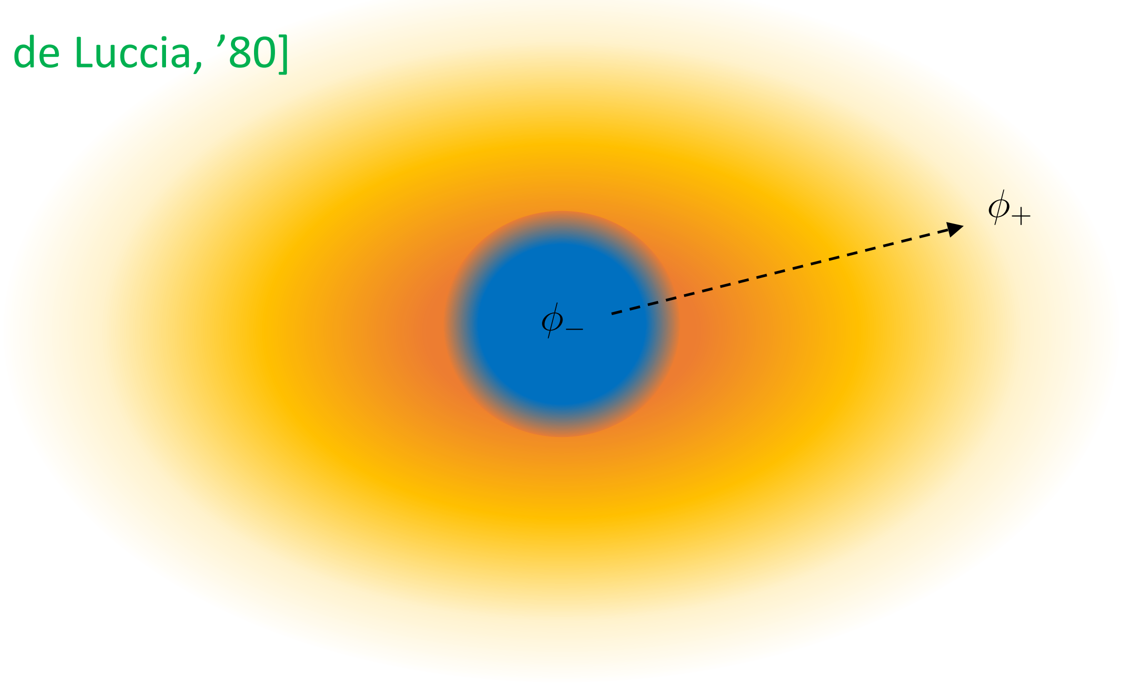
Gravitational domain walls

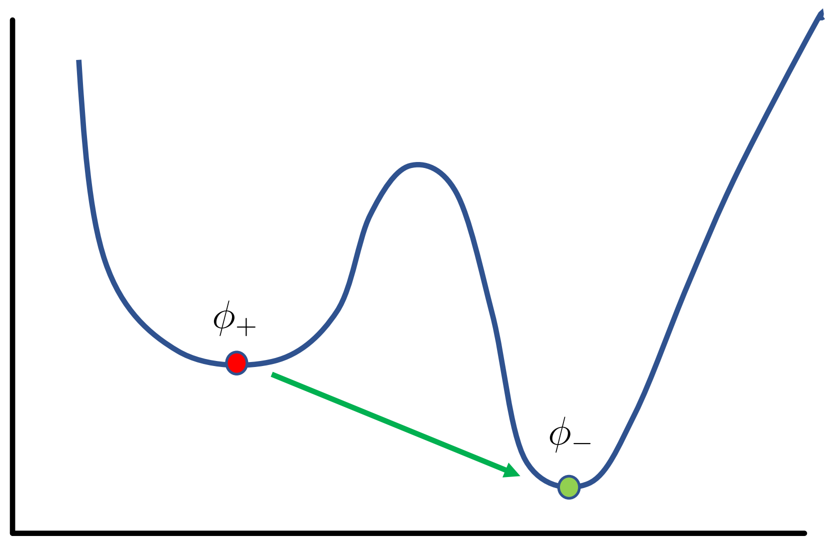
FLRW metric (inflation)

Running solutions: Dynamical Cobordism in String Theory

Coleman-de Luccia tunneling

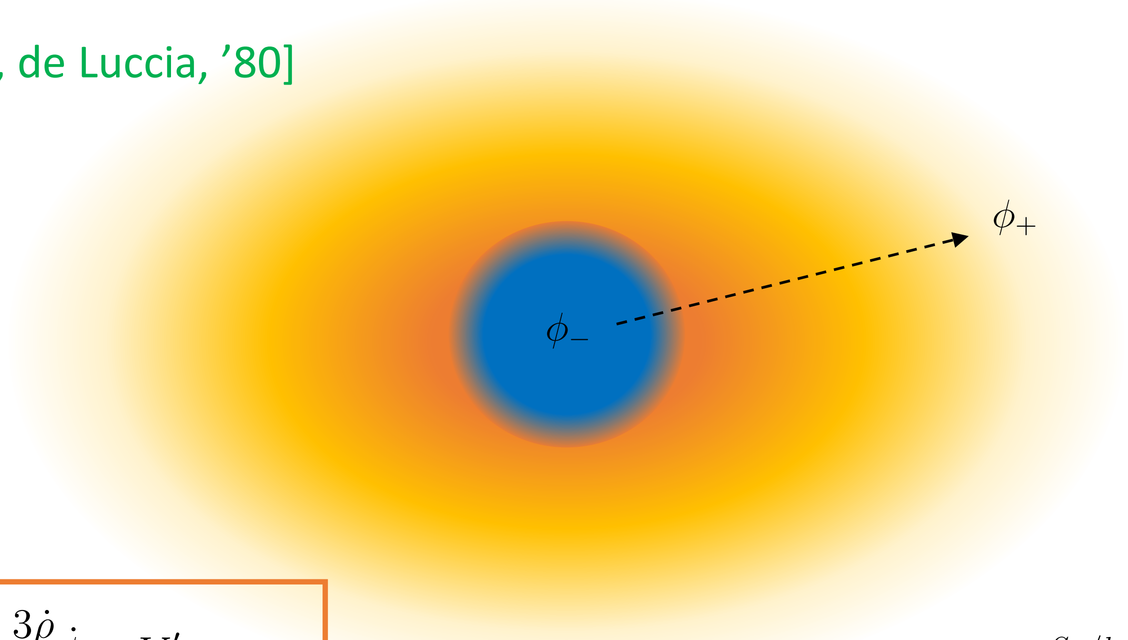
[Coleman, de Luccia, ’80]

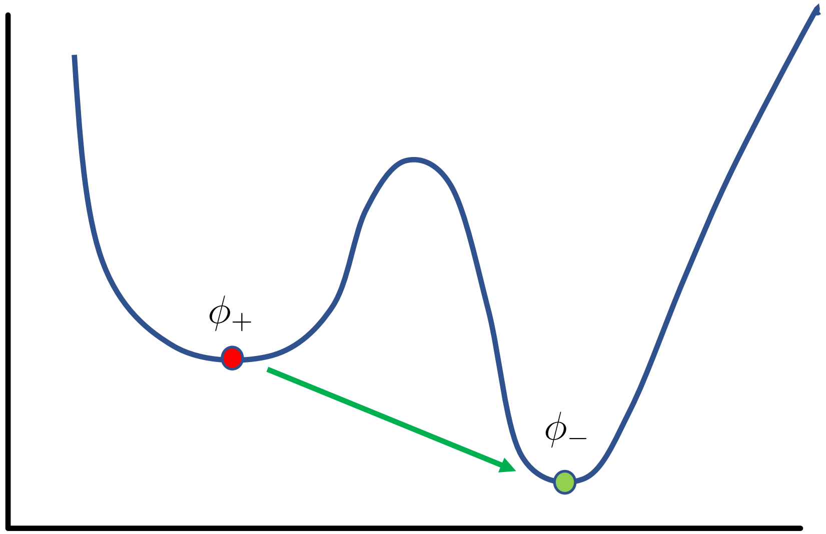




Coleman-de Luccia tunneling

[Coleman, de Luccia, '80]





$$\ddot{\phi} + \frac{3\dot{\rho}}{\rho}\dot{\phi} = V'$$

$$\dot{\rho}^2 = 1 + \frac{\kappa}{3}\rho^2\left(\frac{1}{2}\dot{\phi}^2 - V\right)$$

Tunneling probability: $\Gamma/Vol \propto e^{-S_E/\hbar}$

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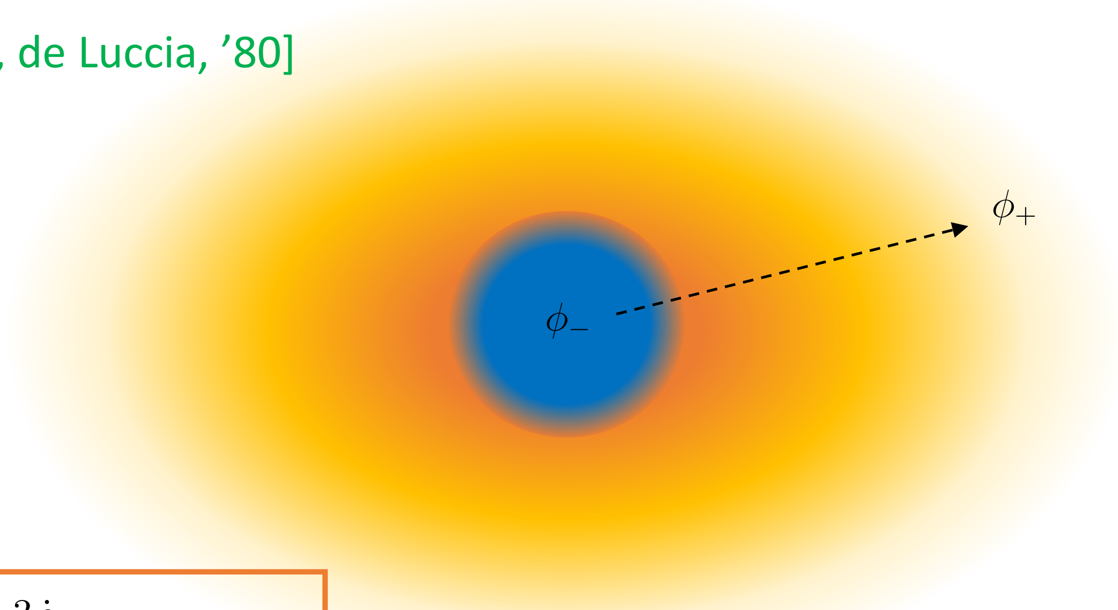

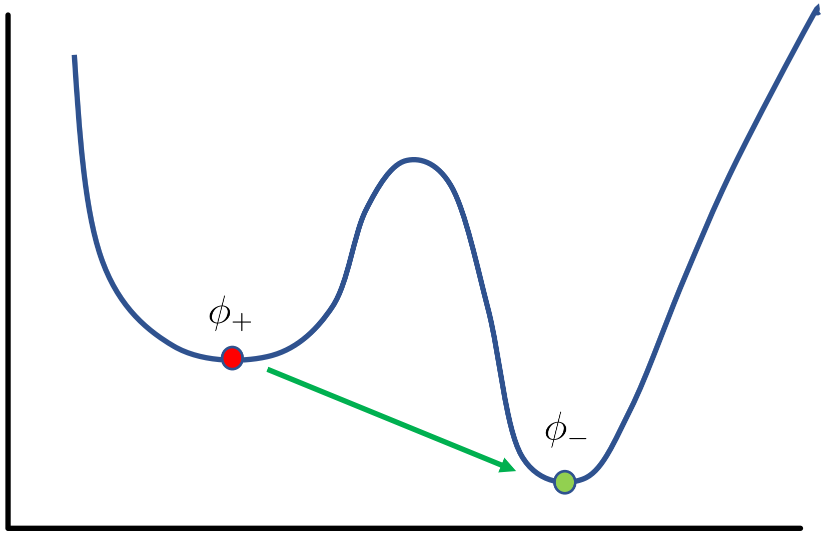

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Tunneling probability: $\Gamma/Vol \propto e^{-S_E/\hbar}$

$$\phi(\xi), \rho(\xi) \longrightarrow V_t(\phi)$$

$$S[V_t] = \frac{6\pi^2}{\kappa^2}\int_{\phi_+}^{\phi_-} d\phi \, \frac{(D + V_t')^2}{V_t^2 D}$$

4D

$\longleftarrow D^2 \equiv V_t'^2 + 6\kappa(V - V_t)V_t$

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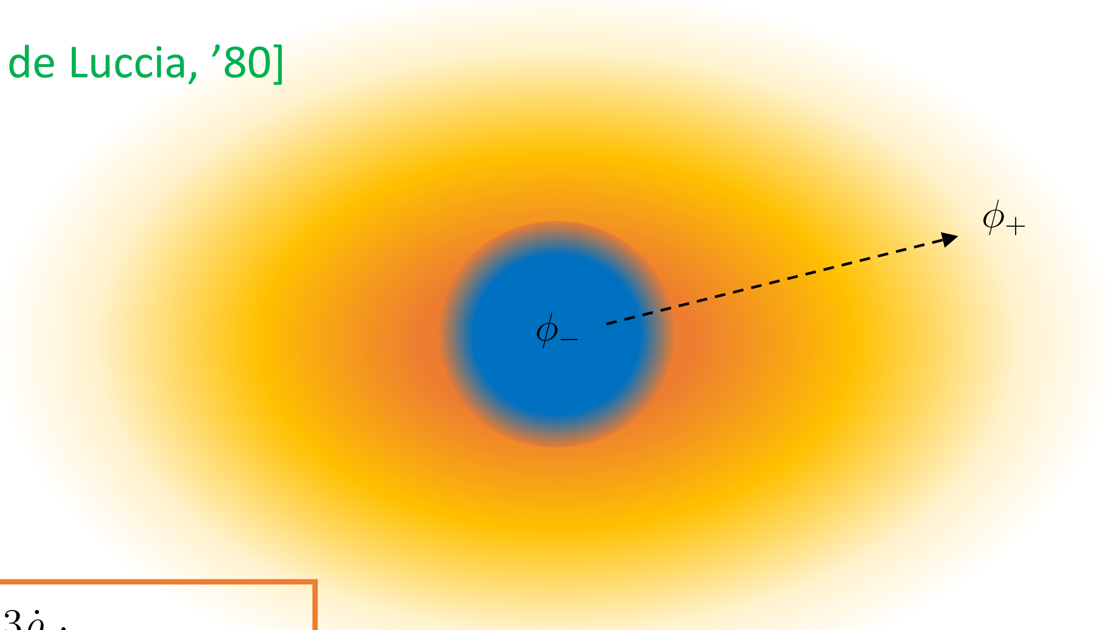

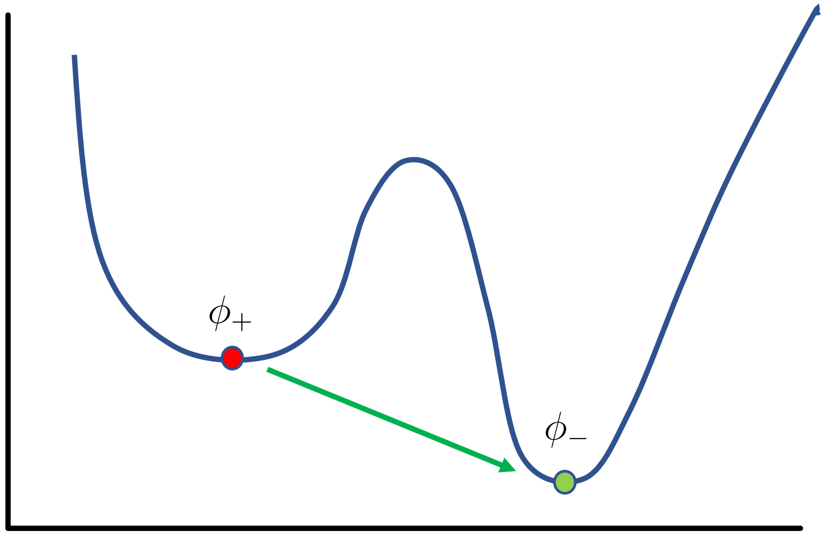

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Encapsulates:

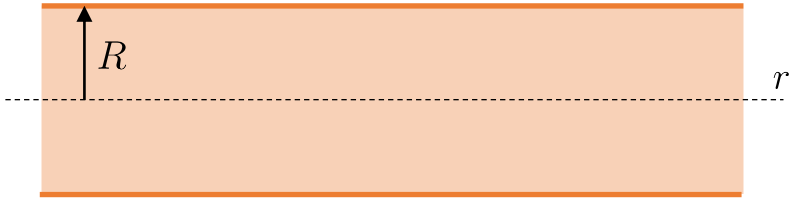
- Gravitational quenching
- Hawking-Moss transitions
- Breitenlohner-Freedman bound

All in the same formula!

Witten’s Bubble of Nothing

[Witten, ’82]

$$ds^2 = d\vec{x}^2 + d\phi_5^2$$



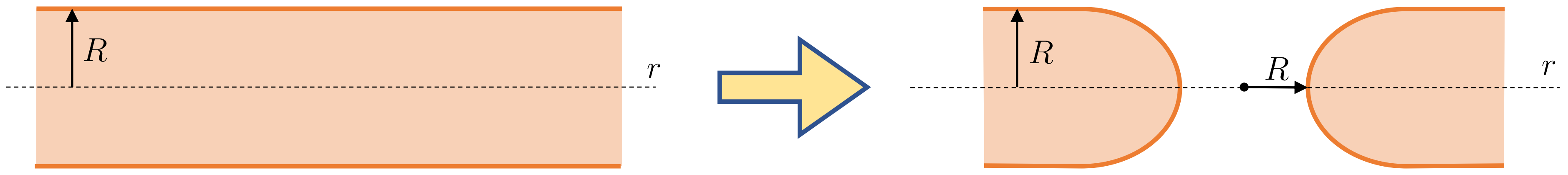
$$\mathbb{R}^4 \times S^1$$

Witten's Bubble of Nothing

[Witten, '82]

$$ds^2 = d\vec{x}^2 + d\phi_5^2$$

$$ds^2 = \frac{dr^2}{1 - R^2/r^2} + r^2 d\Omega_3^2 + \left(1 - R^2/r^2\right) d\phi_5^2$$



$\mathbb{R}^4 \times S^1$

Asymptotically $\mathbb{R}^4 \times S^1$

Near the boundary $\mathbb{R}^2 \times S^3$

$$ds^2 \simeq d\xi^2 + \xi^2 d\phi_5^2 + R^2 d\Omega_3^2$$

Tunneling probability given by $S_E = (\pi m_P R)^2$

Witten's BoN as a CdL tunneling

[Dine, Fox, Gorbatov, '04]

[Draper, García, Lillard, '21]

$$ds_5^2 = e^{\sqrt{\frac{2}{3}}\phi(\xi)}(d\xi^2 + \rho(\xi)^2 d\Omega_3^2) + e^{-2\sqrt{\frac{2}{3}}\phi(\xi)} d\phi_5^2 \qquad S_5 = \int d^5x\,\sqrt{-g_5}\,\frac{R_5}{2}$$

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$$ds_4^2 = d\xi^2 + \rho(\xi)^2 d\Omega_3^2 \qquad S_4 = \int d^4x\, \sqrt{-g_4}\left(\frac{1}{2}R_4 - \frac{1}{2}(\partial\phi)^2\right)$$

No potential!

Radius of ϕ_5 going to zero implies that ϕ goes to infinity

Radius of ϕ_5 going to R implies that ϕ goes to zero

Asymptotic behaviour

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No potential!

Radius of ϕ_5 going to zero implies that ϕ goes to infinity

Radius of ϕ_5 going to R implies that ϕ goes to zero

Asymptotic behaviour

$$(V_t')^2 = \frac{3}{2}V_t[V_t'' - 2V_t] \longrightarrow V_t(\phi) = -\frac{6}{R^2}\sinh^3(\sqrt{2/3}\,\phi)$$

$$S[V_t] = \frac{6\pi^2}{\kappa^2}\int_{\phi_+}^{\phi_-} d\phi\,\frac{(D+V_t')^2}{V_t^2 D}$$

$$S_E = (\pi m_P R)^2$$

Correct result!

Different approaches to Witten's BoN

Comparison between the Euclidean actions

<u>5d solution</u>

$$S_E = -\frac{1}{16\pi G_5}\int d^5x\sqrt{g}R_5 - \frac{1}{8\pi G_5}\int d^4x(K_4 - K_{40})\sqrt{h}$$

$$S_E = 2\pi^2\int_0^\infty d\xi\,\rho^3\left[-\frac{m_P^2}{2}R_4 + \frac{1}{2}\dot{\phi}^2 + \sqrt{\frac{3\kappa}{2}}\left(\ddot{\phi} + \frac{3\dot{\rho}}{\rho}\dot{\phi}\right)\right] - 2\pi^2\rho^2\left(3m_P^2\dot{\rho} + \sqrt{\frac{\kappa}{6}}\rho\dot{\phi}\right)\Bigg|_0^\infty$$

<u>4d KK reduction</u>

Tunneling potential method

$$S_E[V_t] = \frac{6\pi^2}{\kappa^2}\int_0^\infty d\phi\,\frac{(D+V_t')^2}{V_t^2 D} \qquad D^2 \equiv V_t'^2 + 6\kappa(V - V_t)V_t$$

More general BoNs

- Compactification of higher dimensional manifolds
- Introduction of potentials for the fields
- Other type of asymptotic vacuum, such as dS or AdS
- String theory setups: Fluxes, branes on the surface, orientifolds.
  Relation to Dynamical Cobordism.

In the following slides, I will focus on smooth sealing of spacetime on the surface of the bubble

Types of asymptotic decays

In the core of the BoNs, $\phi(\xi = 0) \to \infty$, so $\dfrac{d\phi}{d\xi} \to \infty$ and thus $V_t \to \infty$

$$V_t(\phi) \equiv V(\phi) - \frac{1}{2}\left(\frac{d\phi}{d\xi}\right)^2$$

So we impose $V_t \to \infty$ and solve the EoM $((1+d)V_t' - dV')V_t' = 2d(V_t - V)\left[V_t'' + \dfrac{2\kappa}{d-1}(dV - (d-1)V_t)\right]$

We find an universal behaviour in the limit $\phi(\xi = 0) \to \infty$

Types of asymptotic decays

Type	$V(\phi \to \infty)$	$V_t(\phi \to \infty)$	Constraints	β
A	$V_A e^{a\sqrt{6}\kappa\phi}$	$V_A/(1-a^2)e^{a\sqrt{6}\kappa\phi}$	$V_A > 0\ ,\ a > 1$	$1/(3a^2)$
B	Subleading	$V_{tA}e^{\sqrt{6}\kappa\phi}$	$V_{tA} < 0$	$1/3$
C	$V_A e^{a\sqrt{6}\kappa\phi}$	$V_A/(1-a^2)e^{a\sqrt{6}\kappa\phi}$	$V_A < 0\ ,\ 1/\sqrt{3} < a < 1$	$1/(3a^2)$
D	$V_A e^{a\sqrt{6}\kappa\phi}$	$(3V_A/2)e^{a\sqrt{6}\kappa\phi}$	$V_A < 0\ ,\ a > 1$	1

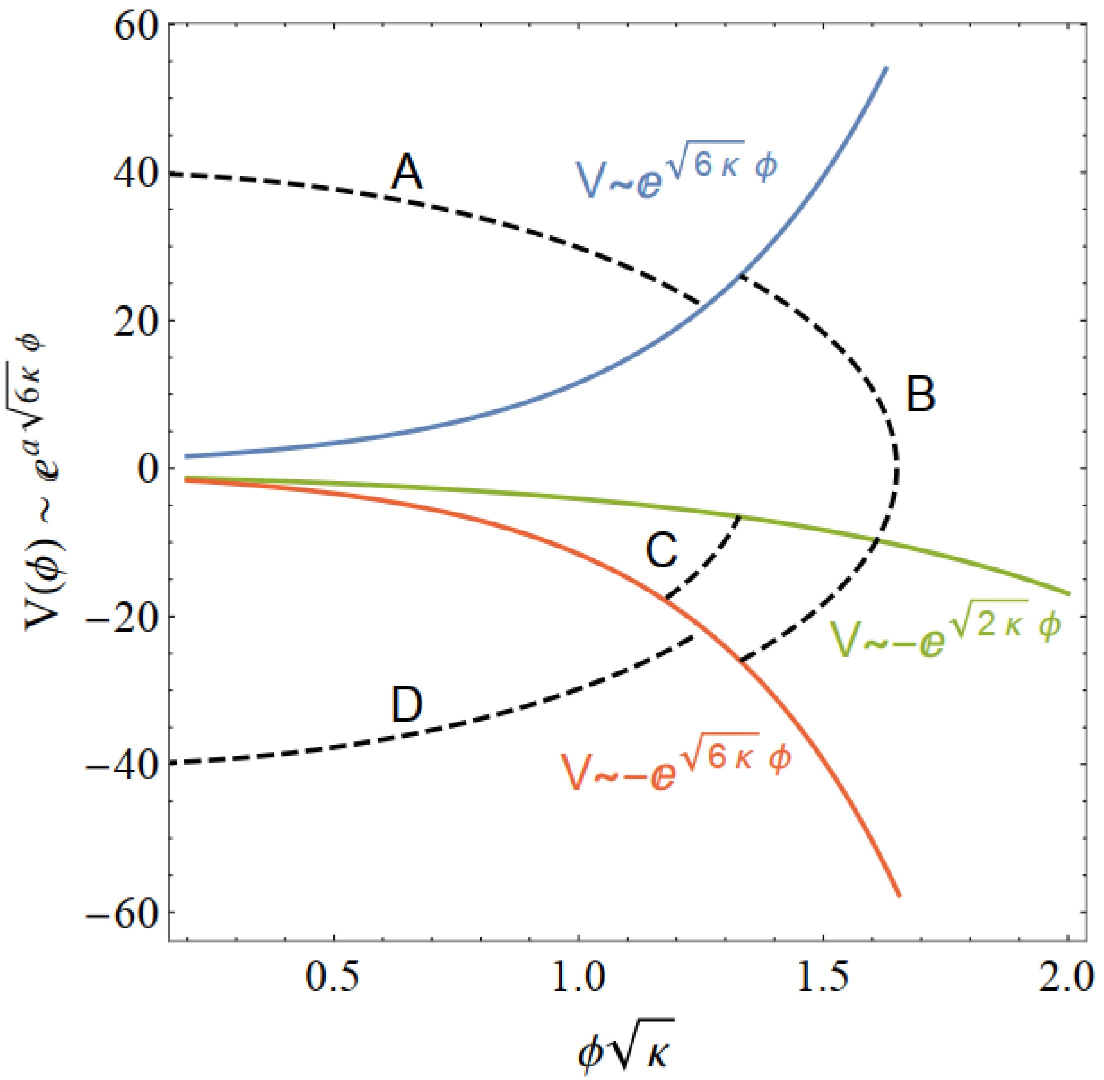
Witten's BoN

Conical singularity

$$ds_4^2 = d\xi^2 + \xi^{2\beta} ds_3^2$$

No flat slices!

Types of asymptotic decays



Example

[Blanco-Pillado, Shlaer, '10]

BoN in $AdS_4 \times S^1$ with the radion stabilized by fluxes

$$S = \int d^5x\sqrt{-G}\left[\frac{1}{2\kappa_5}R(G) - \frac{1}{2}\partial_M\bar{\Phi}\partial^M\bar{\Phi} - \frac{\lambda_5}{4}(\bar{\Phi}\Phi - \eta_5^2)^2 - \Lambda_5\right]$$

$$\Phi(x^M) = f_5(r)e^{iny}$$

Scalar field

$$ds^2 = dr^2 + B^2(r)(-dt^2 + \cosh^2 t\, d\Omega_2^2) + r^2C^2(r)dy^2$$

This solution behaves in the same way that Witten's BoN near the core of the bubble (type B)

Asymptotic behaviour: $f(r=0) = 0, \quad f(r=\infty) = f_\infty$

To make the solution well defined at the bubble

The solution was interpreted to be a de Sitter solitonic 2-brane in a 5d AdS spacetime

Numerical solution gives a finite action!

Example

As we did with Witten's BoN, we can now compactify the solution to 4d: $rC(r) = L\exp(-\sqrt{2/3}\phi)$

$$S = \int d^4x\sqrt{-g}\left\{\frac{R(g)}{2} - \frac{(\partial\phi)^2}{2} - \frac{(\partial f)^2}{2} - V(\phi, f)\right\}$$

$$V(\phi, f) = e^{\sqrt{2/3}\phi}\left[\frac{n^2}{2L^2}f^2e^{2\sqrt{2/3}\phi} + \frac{\lambda}{4}(f^2-\eta^2)^2 + \Lambda\right]$$

$$f \equiv \sqrt{2\pi L}f_5\,, \quad \eta \equiv \sqrt{2\pi L}\eta_5\,, \quad \Lambda \equiv 2\pi L\Lambda_5\,, \quad \lambda \equiv \lambda_5/(2\pi L)$$

Asymptotic behaviour: $\phi(r=0) = \infty, \quad \phi(r=\infty) = 0$

Multifield solution

$$V_t(\varphi) = V - \frac{1}{2}\dot{\varphi}^2 \qquad d\varphi^2 \equiv d\phi^2 + df^2$$

$$((1+d)V_t' - dV')V_t' = 2d(V_t - V)\left[V_t'' + \frac{2\kappa}{d-1}(dV - (d-1)V_t)\right] \qquad 2(V - V_t)\phi'' = \partial V/\partial\phi - \phi' V'$$

New!
(Multifield)

Near $r = 0$ →

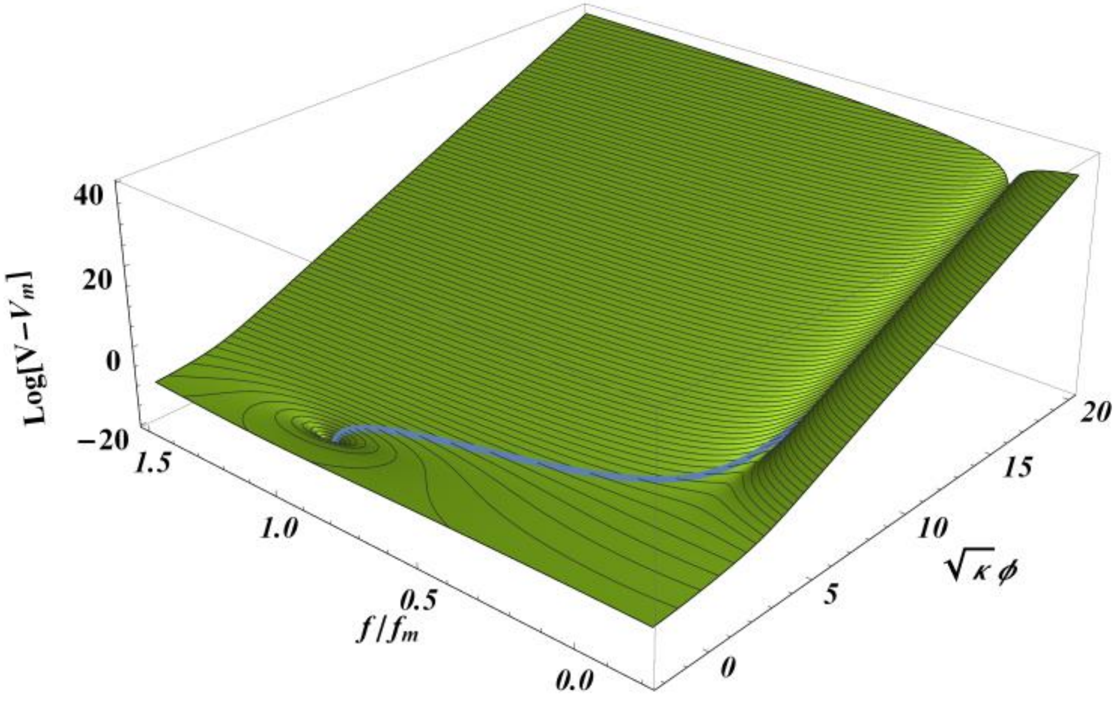
$$f \simeq f_0 e^{-\sqrt{2/3}\varphi} \qquad \phi \simeq \varphi$$

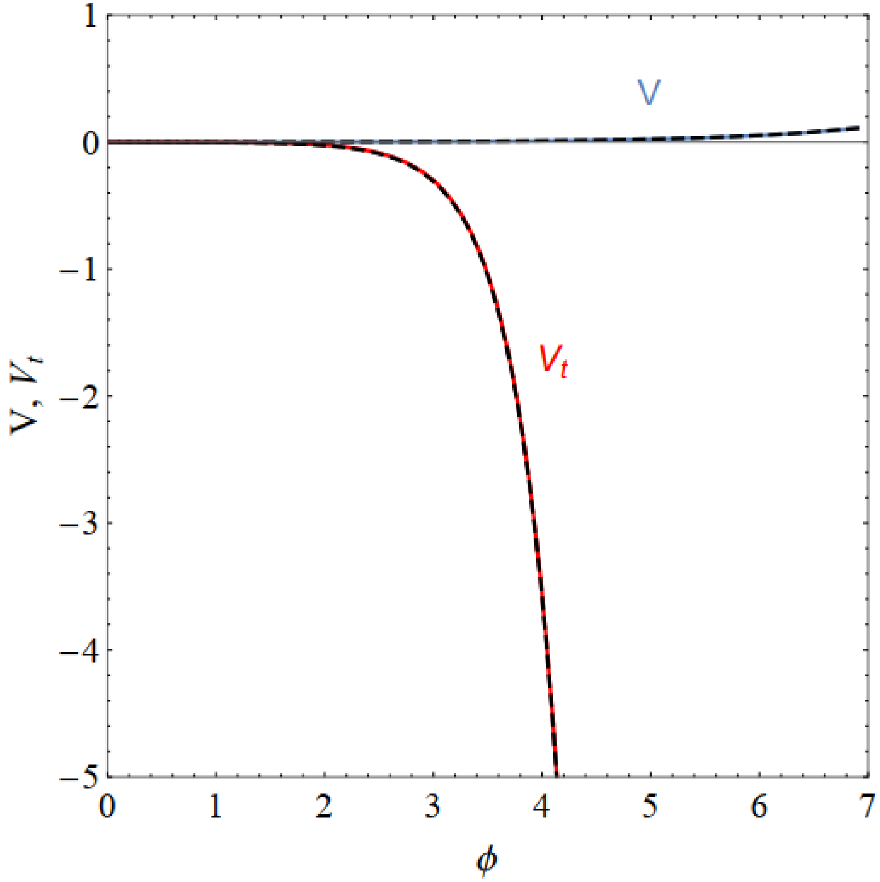
↓

$$V \simeq \left(\Lambda + \frac{1}{4}\lambda\eta^4 + \frac{n^2 f_0^2}{2L^2}\right) e^{\sqrt{2/3}\phi} , \quad V_t \simeq -\frac{3n^2}{4L^2} e^{\sqrt{6}\phi}$$

(Type B)

Multifield solution





Final remarks

- The Tunneling Potential Method is a powerful way to study universal properties of general Bubbles of Nothing
- It also makes easier the finding of analytical and numerical solutions
- Makes clear the relation between Bubbles of Nothing and ETW branes in String Theory Dynamical Cobordism

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Pending questions:

- Investigate the nature of the conical singularity solutions (cannot be obtained by a smooth bubble boundary)
- With the same asymptotic potential V, what type is the preferred decay channel?
- In a landscape with various minima, what decay is more probable, CdL, HM or BoN?

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Work in progress, so comments/questions are welcome

Thank you!